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*On the Comparison of various Tables of Annuities.** *By* J. W. LUBBOCK, ESQ. (*now* SIR J. W. LUBBOCK, BART.), B.A., F.R. & L.S., *of Trinity College, Cambridge.*

[Extracted, by permission of the Author, from the *Transactions of the Cambridge Philosophical Society.*]

1. A SHORT time back I transmitted to the Philosophical Society of Cambridge some remarks upon the construction of tables of annuities: my object in that paper was to show how the probabilities upon which annuities depend should be deduced from tables of mortality, and I gave in illustration some tables of annuities calculated from observations of the mortality at Chester, by Dr. Haygarth, which appear to have been made with very great care. I have since compared these tables with a great many others, and I now present the result of this comparison.

2. Very few registers of mortality give the deaths at every year throughout life; they generally give the deaths between birth and 5, 5 and 10, 10 and 20, 20 and 30, and so on for every decade. When the deaths are given between birth and 5, the *living* at 5, at 20, 30, &c., are known, and in order to form a complete table of mortality it is necessary to interpolate the number of living at each intermediate age.

If the probability of an individual aged m years living n years be called $p_{m,n}$, if r is the rate of interest, and if the same hypo-

* This is the paper referred to in the note at page 205, vol. iv., of this *Journal.*

thesis of probability be adopted as in my former paper, which amounts to increasing by 1 the deaths at every age,

$$p_{m,\,n}=\frac{\text{living at } n+101-n}{\text{living at } m+101-m},$$

the value of a payment of unity after n years is $\frac{p_{m,\,n}}{(1+r)^n}$, and the value of an annuity is $\Sigma\left\{\frac{p_{m,\,n}}{(1+r)^n}\right\}$, m being constant in this expression, and n variable.

Instead however of interpolating values of $p_{m,\,n}$ between those values which are known, it is better to interpolate at once between the values of $p_{m,\,n}\times(1+r)^{-n}$ which are given; but even this labour is unnecessary, because $\Sigma\frac{p_{m,\,n}}{(1+r)^n}$, or the value of the annuity, is a function of those terms only of the series which are given.

Let y_0, y_i, y_{2i}, . . . y_{ni}, $y_{(n+1)i}$, &c., be successive values of any variable y,

$$\begin{aligned}
y_0 &= y_0,\\
y_i &= y_0+i\Delta y_0+\frac{i.i-1}{1.2}\Delta^2 y_0+, \text{\&c.},\\
y_{2i} &= y_0+2i\Delta y_0+\frac{2i.2i-1}{1.2}\Delta^2 y_0+, \text{\&c.},\\
y_{ni} &= y_{ni},\\
y_{(n+1)i} &= y_{ni}+i\Delta y_{ni}+\frac{i.i-1}{1.2}\Delta^2 y_{ni}+, \text{\&c.},
\end{aligned}$$

$$y_0+y_i+y_{2i}+y_{3i}\ldots\ldots+y_m+y_{(n+1)i}+, \text{\&c.}+y_{(mni-1)}$$
$$=n(y_0+y_{ni}+y_{2ni}\ldots\ldots\ldots\ldots+y)$$
$$+i\{1+2+3\ldots\ldots\ldots n-1.\}\{\Delta y_0+\Delta y_{ni}+, \text{\&c.}+\Delta y_{(m-1)ni}\}$$
$$+\frac{i}{1.2}\{1.i-1+2.i-2+, \text{\&c.}+n-1.1\}\{\Delta^2 y_0+\Delta^2 y_{ni}+, \text{\&c.}\},$$

$$\begin{aligned}
\Delta y_0+\Delta y_{ni}+, \text{\&c.}+\Delta y_{(m-1)ni} &= y_{mni}-y_0,\\
\Delta^2 y_0+\Delta^2 y_m\ldots\ldots+\Delta^2 y_{(m-1)ni} &= \Delta y_{mni}-\Delta y_0,\\
\Delta^3 y_0+\Delta^3 y_m\ldots\ldots+\Delta^3 y_{(m-1)ni} &= \Delta^2 y_{mni}-\Delta^2 y_0.
\end{aligned}$$

When $ni=1$, $i=\frac{1}{n}$, the sum of the series is equal to

$$n(y_0+y_1+y_2, \text{\&c.}\ldots\ldots+y_{m-1})+\frac{1}{n}\{1+2+3\ldots\ldots+n-1\}\{y_m-y_0\}$$
$$-\frac{1}{1.2n^2}\{1.n-1+2.n-2+3.n-3\ldots\ldots+n-1.1\}\{\Delta y_m-\Delta y_0\}$$
$$+\frac{1}{1.2.3n^3}\{1.n-1.2n-1+2.n-2.2n-2\ldots\ldots$$
$$+n-1.1.n+1\}\{\Delta^2 y_m-\Delta y_0\}+, \text{\&c.}$$

The coefficient of $\Delta^q y_m-\Delta^q y_0$ is equal to the coefficient of x^{q-1}

in the development of $\dfrac{(1+x)^{ni}-(1+x)}{1-(1+x)^i}$: or, in other words, if this coefficient be called z_q, $\dfrac{x\{(1+x)^{ni}-(1+x)\}}{1-(1+x)^i}$ is the generating function of z_q; and since $ni=1$,

$$\frac{(1+x)^{ni}-(1+x)}{1-(1+x)^i}=\frac{x}{(1+x)^i-1}$$

$$=\frac{1}{i}-\frac{i-1}{2i}x+\frac{i-1.i+1}{12i}x^2-\frac{i-1.i+1}{24i}x^3+, \text{\&c.}$$

$$=n+\frac{n-1}{2n}x-\frac{n-1.n+1}{12n}x^2+\frac{n-1.n+1}{24n}x^3+, \text{\&c.}$$

The sum of the series is

$$n(y_0+y_1+y_2, \text{\&c.}+y_{m-1})+\frac{n-1}{2}\{y_m-y_0\}-\frac{n-1.n+1}{12.n}\{\Delta y_m-\Delta y_0\}$$
$$+\frac{n-1.n+1}{24.n}\{\Delta^2 y_m-\Delta^2 y_0\}+, \text{\&c.}$$

Laplace has given (in the fourth volume of the *Mécanique Céleste,* p. 206) the particular value of this series which obtains when the interval i which separates the values of y is indefinitely diminished.

In this case the coefficient of $\Delta^q y_m-\Delta^q y_0$ is found by integrating

$$\frac{i.(i-1)(i-2)\ldots\ldots\ldots.(i-q)\,di}{1.2.3\ldots\ldots\ldots\ldots q+1}$$

from $i=0$ to $i=n$, if $n=1$, the sum of the values of y or the area of the curve between y_0 and y_m,

$$=\tfrac{1}{2}y_0+y_1+y_2\ldots\ldots\ldots+\tfrac{1}{2}y_m-\frac{1}{12}\{\Delta y_m-\Delta y_0\}+\frac{1}{24}\{\Delta^2 y_m-\Delta^2 y_0\}.$$

In applications of the former series to the calculation of annuities, reversionary payments, &c., y_m, Δy_m, $\Delta^2 y_m$, &c., $\doteq 0$.

The first term in the series of the values of y or y_0 is the value of a present payment $=1$, if we neglect the term

$$\frac{n-1.n+1}{12.n}\{\Delta y_m-\Delta y_0\}$$

and the following, and suppose the values of the annual payments to be in arithmetical progression, the value of an annuity on the life of a person aged 20 to commence at the end of the first year.

If $n=10$, $y_0=1$, $y_1=\dfrac{p_{20,\,10}}{(1+r)^{10}}$,

$$=10\left\{1+\frac{p_{20,10}}{(1+r)^{10}}+\frac{p_{20,20}}{(1+r)^{20}}+,\&c.\right\}-\frac{9}{2}-1,$$

$$=10\left\{\frac{p_{20,10}}{(1+r)^{10}}+\frac{p_{20,20}}{(1+r)^{20}}+,\&c.\right\}+\frac{9}{2},$$

the values of annuities at 0, 5, &c. may be obtained in a similar manner. This value of the annuity will be a very close approximation: the error, whatever it be, will be nearly constant for different tables of mortality; and as the first correction, which is in this case $\frac{9}{2}$, is constant, the whole correction may be considered as constant. It may therefore be determined easily by calculating the annuity first accurately, and afterwards by the approximate method from any table of mortality in which the deaths are given for every age: the difference between the two values so obtained will be the correction required. By means of the Chester Table for Males, I determined the correction as follows, supposing the table of mortality to give the living at 0, 5, 10, 20, 30, 40, 50, &c., and that the annuity commences at the end of the first year.

Age.	Value.	Age.	Value.
At Birth	2·481	30	4·109
5	3·692	40	4·024
10	4·167	50	3·920
20	4·242	60	3·792

Thus the value of an annuity at 20 is

$$10\times\left\{\frac{p_{20,10}}{(1+r)^{10}}+\frac{p_{20,20}}{(1+r)^{20}}\ldots\ldots\&c.\right\}+4{\cdot}242.$$

How close an approximation this method gives may be seen in Table II.,* where I have placed underneath the results which I have obtained those which have been obtained by other writers. The same series shows that the value of an annuity of £1 paid half yearly is the value of the same annuity paid yearly $+\frac{1}{4}$, and the value of an annuity of £1 paid weekly is the value of the same annuity paid yearly $+\frac{51}{104}$; the annuity being supposed to commence at the end of a year, and the first weekly payment to commence at the end of a week.

When the table of mortality which is made use of gives the deaths at every age, the preceding method can only be considered as an approximation; but in all cases I believe the error due to this method will be less than the error due to the errors of the observations.

* *See* page 292, and the note there.

The same series furnishes a method, which I think is the simplest which can be proposed, of calculating approximately the values of annuities or insurances on two or three lives.

The value of an insurance on one life is easily deduced from the value of the annuity; in fact, if A is the value of the annuity, the value of the insurance in a single payment is

$$\{1+\mathrm{A}\}\frac{1}{1+r}-\mathrm{A},$$

and the value of the premium is

$$\frac{1}{\mathrm{A}(1+r)}+\frac{1}{1+r}-1.$$

When the persons observed, upon whom the table of mortality is founded, are few in number, and the deaths are given for every year, they will present considerable irregularities, owing partly to the effect of accidental causes, and partly to the unavoidable errors of the observations; but these causes may be considered in theory as identical. If e be the probability that an individual died in the year in which he is recorded to have died, e_1 the year after, e_n the n^{th} year after, &c., and if the table of mortality be founded upon a population observed from birth throughout life, upon the same hypothesis of probability, *à priori* as before, the formula which I gave in my former paper on this subject* shows that, if d_n be the number of deaths recorded to have taken place at the n^{th} age, the probability at the birth of a child that he will die at the n^{th} age is

$$\frac{\Sigma\{d_{n+m}\times e_m\}+1}{\Sigma d_n+p};$$

Σd_n being the total number of persons observed, and p the number of cases possible, or ages at which deaths are supposed to take place. The values of e are to a certain extent arbitrary. If e be supposed to be constant and $=\dfrac{1}{m+1}$, and that the values of e are $e_{-\frac{m}{2}}\ldots e_{-1},\ e,\ e_1\ldots e_{\frac{m}{2}}$, this amounts to taking the mean of the deaths which are recorded to have taken place within $\dfrac{m}{2}$ years of the age n. Generally, however e be supposed to vary, $\Sigma e_m=1$. This theory shows how a table of mortality should be corrected, for the irregularities which present themselves, when the observations are not numerous.

The number p may also be considered as arbitrary; and by

* *See* page 197.

altering this, which amounts to increasing the deaths at every age by an arbitrary quantity, the table may also be corrected: but the former method is simpler.

With the assistance of Mr. Deacon, I have calculated the tables of annuities at the end of this paper * by the approximate method given above, and the data or table of observations from which they are taken is prefixed to each.

3. Table I. contains different registers of mortality, giving first the actual number of living deduced from the recorded deaths, and then the same reduced to the radix 1000.

The table for Paris is taken from the *Annuaire du Bureau des Longitudes.*

The Breslau Table is taken from Dr. Halley's paper in the *Transactions of the Royal Society;* it was formed from observations communicated to the Royal Society by Mr. Justell. Dr. Halley has not given the observations themselves.

Kerseboom's Table was formed by him from registers of life annuitants in Holland and West Friesland—Desparcieux's tables, from lists of the nominees in the French Tontines. These two must be considered as formed upon very select life.

The tables for Brussels and Amsterdam are taken from the *Recherches sur la Population dans le Royaume des Pays Bas,* by M. Quetelet.

The table for Sweden was formed "from observations of the proportion of the living to the numbers who died at all ages for 21 years, from 1755 to 1776, in the kingdom of Sweden" (*see* Dr. Price, vol. ii. p. 140). The table for Montpelier is from a Memoir by Mr. Morgue, in the first volume of the *Mémoires de l'Institut.* The Northampton Table is taken from the deaths in All Saints' Parish, Northampton, from 1735 to 1780 (*see* Dr. Price, vol. ii. p. 95). The Carlisle Table of Mortality, as given by Mr. Milne, was formed by him from the observations of the mortality which are given in the next column, combined with two enumerations of the population. The numbers upon which this table is formed are very small. The expectation of life is given at the foot, calculated from each by a method similar to that I have explained for calculating annuities.

Table II. contains annuities deduced from the preceding.

Table III. contains tables of mortality in which the sexes are distinguished, and Table IV. contains annuities deduced from them. It will be observed, that all these tables agree in giving to

* *See* page 292, and the note there.

females a greater longevity than to males; a fact which is further confirmed by the circumstance that in all countries—with the exception, I believe, of Russia—notwithstanding the male births exceed the female, the number of females in the population exceeds that of the males.

Mr. Griffith Davies has published tables of annuities taken from statements of Mr. Morgan in his addresses to the general courts of the Equitable Society, and in notes added by him to the latter editions of Dr. Price's *Observations on Reversionary Payments.* In Mr. Morgan's address to the general court held on the 24th of April, 1800, he stated that the decrements of life among the members of the Equitable, for the preceding 30 years, had been, to those of the Northampton—

From	10	to	20,	as	1 : 2
„	20	„	30	„	1 : 2
„	30	„	40	„	3 : 5
„	40	„	50	„	3 : 5
„	50	„	60	„	5 : 7
„	60	„	80	„	4 : 5

which statement is confirmed in his subsequent addresses.

In a recent publication, Mr. Morgan admits that he was not then aware of the great number of instances in which there are several policies on one and the same life, and he says that this circumstance very materially affects Mr. Davies's calculations.

Such statements as these appear to me too vague to be made the basis of calculations, although the experience of the Equitable Society would be most valuable, if we were acquainted with all the details concerning it.

Mr. Finlaison has very recently published extensive tables of mortality formed from the Government Tontines and Annuitants, which are rendered equally valuable by the accuracy of the materials from which they have been deduced, and the very great care and attention which has been bestowed on them by the author. Mr. Finlaison has done me the favour to prepare for me a summary of these tables, which is to be found in Table V. in a form in which it may be easily compared with the other tables which I have given.

Mr. Finlaison, in his report to the Lords of the Treasury, explains at length the manner in which he made use of the records of the Tontines. Mr. Finlaison observes, that "the facts shown in these observations bear conclusive testimony that the rate of mortality in England has, during the last century, diminished in a

very important degree, on each sex equally, but not by equal gradations, nor equally at all periods of life; and that while, in regard to the males, it seems in early and middling life to have remained for a long time as it stood about fifty years ago, in respect of the females it has during the same time visibly and progressively diminished, to this day, by slight but still sensible gradations." This fact is at variance with the opinion that the improvement which has taken place in life is to be attributed to the introduction of vaccination. Epidemics, however, are of much less frequent occurrence in England than they were formerly, which circumstance must tend materially to diminish the rate of mortality.

The great plague years in London were 1592, 1593, 1603, 1625, 1636 and 1665, in which the burials were as follows:

	1592.	1593.	1603.	1625.	1636.	1665.
Total Deaths	25,886	17,844	37,294	51,758	23,359	97,306
Deaths of the Plague	11,503	10,662	30,561	35,417	10,460	68,596

Now the average number of deaths in London is about 20,000, and the actual number varies very little.

Observations such as those presented by Mr. Finlaison, where the deaths are given at every age, are particularly well calculated to determine delicate points, such as any small increase of the rate of mortality at different ages. A small increase of mortality, according to Mr. Finlaison's tables, takes place about 23; thus, in observation 19 (p. 56 of Mr. Finlaison's report), it appears that there is a minimum of mortality at 13, a maximum at 23, and a minimum again at 33. This does not obtain in Mr. Finlaison's observations on females. It is very remarkable, that the same circumstance is to be observed in the Chester Tables, though here it is found equally in the tables for males and females: this appears to me a great proof of their accuracy, and of the fidelity with which Dr. Haygarth recorded the facts which were presented to him. Dr. Price says—"the bills (for Northampton) give the numbers dying annually between 20 and 30 greater than between 30 and 40; but this being a circumstance which does not exist in any other register of mortality, and undoubtedly owing to some accident and local causes, *the decrements were made equal between* 20 *and* 40," &c. (vol. ii. p. 97.)

However accurate the observations be upon which Mr. Finlaison's results are founded, it must be recollected that the lives were selected from a selected class; and it remains to be shown that the mortality in the lower classes of society is the same as in the higher, and that selection produces no effect on the results.

4. Tables of mortality which are founded upon registers of deaths only are subject to an error arising from the supposition that the population is stationary, as was long ago noticed by Dr. Price (vol. ii. p. 251).

The probability of an individual dying in a given n^{th} year of his life, if the effect of migration be neglected, is the number of deaths of that age divided by the number of births in one year, n years previously—which, if the population were stationary, would be the same as the total of deaths in any year.

If therefore the births, n years previously, are $\gtrless$ than the total of deaths at all ages in the year of the observation, the probability of an individual dying at the n^{th} age is $\lessgtr$ than the quotient of the deaths at that age divided by the total of the deaths at all ages. In America this effect is, I think, clearly perceptible, and has led some persons to conclude that the population in that continent is more unhealthy than in Europe.

The following table has been formed from the bills of mortality for Boston, New York, Philadelphia, and Baltimore, in 1820—

Age.	Living.	Age.	Living.
0	1000	40	254
5	587	50	160
10	549	60	96
20	495	70	53
30	371	80	24
Expectation of life at birth, 24·959.			

which table is much lower than any of the others; but the annual rate of increase of the population in the United States, between 1810 and 1820, was about 1·034. In England, at the same time, it was only 1·016. In order to show directly the effect which an increase in the population produces in the table of mortality, I have calculated three tables from the Chester tables of mortality, supposing the deaths at the time of the observation to be equal to

the deaths 40 years previously (which was nearly the case in this country in the last century), and the births to increase annually in a geometrical progression of which the common ratio is given.

The column	A	supposes the ratio	of increase	to be	1·005
,,	B	,,	,,	,,	1·010
,,	C	,,	,,	,,	1·015

The column D is calculated in the same way for females, and supposes the ratio of increase to be 1·005. The ratio 1·005 is very nearly what obtained in England during the last century, according to the Parliamentary Reports. The births in all England, in the year 1700, were 138,979, and in 1780, 201,310, making the mean annual rate of increase 1·0046. In the county of Chester, taken by itself, in 1700 they were 2,690, and in 1780, 4,592, making the mean annual rate of increase 1·0061; therefore the columns A and D, which I have given at length in Table VII., must approach very nearly to exactitude: and, considering attentively the limits of the errors of which observations of this kind are susceptible, I think that it is improbable that the longevity in this country generally, when the Chester Table was formed, was quite so great as that indicated by Mr. Finlaison's tables and the experience of the Equitable Society. It may have improved since.

When the law of mortality in any country, and the number of births in each year during the century previous to any given epoch, are known, it is easy to assign the total number of persons living at every age; for if $p_{0,n}$ be the probability of a child at birth surviving n years, b_n the births n years previously, the number of living in the population at the n^{th} age is $p_{0,n} \times b_n$, and the ratio of the living at that age to the whole population is

$$\frac{p_{0,n} \times b_n}{\Sigma(p_{0,n} \times b_n)}.$$

I have calculated Tables VIII. and IX. in order to show the effect which is produced by a given increase of the births. Table VIII. shows the proportion of the living at each age, and of the deaths to the whole population, when the law of mortality obtains which is given by Table VII. The male births are supposed to be to the females as 104 to 100. Table IX. is calculated upon the supposition that the law of mortality obtains which is given by the Carlisle Table in Mr. Milne's work (vol. ii. p. 564). The following are the results which are given by these tables:—

Ratio of Increase of the Births yearly	1		1·005		1·010		1·015		1·020	
	Chester.	Carlisle.	Chester.	Carlisle.	Chester.	Carlisle.	Chester.	Carlisle.	Chester.	Carlisle.
Ratio of the Births to the Population	$\frac{1}{37·381}$	$\frac{1}{39·219}$	$\frac{1}{32·092}$	$\frac{1}{33·783}$	$\frac{1}{27·964}$	$\frac{1}{29·274}$	$\frac{1}{24·660}$	$\frac{1}{25·654}$	$\frac{1}{21·939}$	$\frac{1}{22·722}$
Ratio of the Deaths to the Population	$\frac{1}{37·381}$	$\frac{1}{39·219}$	$\frac{1}{38·417}$	$\frac{1}{39·440}$	$\frac{1}{38·409}$	$\frac{1}{40·054}$	$\frac{1}{38·544}$	$\frac{1}{40·085}$	$\frac{1}{38·187}$	$\frac{1}{39·781}$
Ratio of Increase of the Population yearly	..	..	1·005	1·005	1·010	1·010	1·015	1·015	1·020	1·020
Population doubles in	..	..	138 yrs.	138 yrs.	69 yrs.	69 yrs.	46 yrs.	46 yrs.	35 yrs.	35 yrs.
Deaths are equal to the Births after ..	..	..	36 yrs.	31 yrs.	33 yrs.	31 yrs.	30 yrs.	30 yrs.	27 yrs.	28 yrs.

The ratio of the deaths to the population is nearly constant, according to both these tables, whatever be the rate of increase of the births; when the ratio of the births to the population is constant, the rate of increase of the population is necessarily the same as that of the births. The rate of increase of the births has been supposed to be constant—a small inequality in this rate, unless it be of long period, will not produce any sensible difference in the results; but, although the total number of deaths which take place in a given population is not much influenced by the rate of increase, the apparent table of mortality is much altered. In order to show the extent of the error which is likely to arise from this circumstance, I have given the apparent tables of mortality corresponding to each rate of increase of the births.

According to Mr. Rickman, in the *Population Abstract,* 1821, the ratio of the deaths to the population in England, at that time, was 1 to 57. This ratio is considerably less than would be given by any table of mortality; and it is probable, therefore, that the number of unentered burials is much greater than Mr. Rickman has supposed. Since the ratio of the deaths to the population is nearly constant when the law of mortality is given, this rate would be an excellent criterion of the longevity of different countries, if it could be accurately ascertained; to this, however, many difficulties are opposed.

In the Tables VIII. and IX. the rate of increase of the births is arbitrary. In order to see how far the mortality in this country coincides with that given by Table VII., I have formed Table X., taking the values of p from that table, and supposing the births in the century previous to 1821 to have been the same as the christenings that are given in the *Population Abstract* before referred

to; and since the ratio $\frac{p_{0,n} \times b_n}{\Sigma\,(p_{0,n} \times b_n)}$ involves only the ratios of the births, which must be nearly the same as the ratios of the christenings, the error introduced by this hypothesis is altogether insensible.

I have placed, for the sake of comparison, the results given by the census of 1821 with the results deduced from theory; and they agree, I think, within the limits of the errors of which the census is susceptible, and much nearer than the results of different counties agree with each other. The number of deaths in a population of 1,000 males and females, according to the law of mortality of Table VII., is 271, making the ratio of the deaths to the population about $\frac{1}{37}$. Calculating the deaths between 0 and 5, to which period Mr. Finlaison's observations do not extend, from the same table, and those at the succeeding ages from Mr. Finlaison's observations 11 and 19, the total number of deaths which results in a population of 1,000 males and females is 244, nearly, and the ratio of the deaths to the population about 1 to 41: which is far greater than the ratio given by Mr. Rickman.

The following are some of the elements of the population of England and France. Those for England are deduced from the returns in the *Population Abstract* of 1821, before referred to; and those for France, from the *Annuaire du Bureau des Longitudes* for 1829.

	England.	France.
Ratio of males to females	·95764 : 1	
„ male births to female	1·0435 : 1	1·0656 : 1
„ „ deaths to female	1·0024 : 1	1·0180 : 1
„ „ legitimate births to female	.. ..	1·06795 : 1
„ „ illegitimate births to female	.. ..	1·04844 : 1
„ population to marriages in one year	122·50 : 1	132·619 : 1
„ „ births in one year	32·274 : 1	31·535 : 1
„ „ deaths in one year	54·296 : 1	39·423 : 1
„ births to marriages	3·5902 : 1	4·205 : 1
„ legitimate births to marriages	.. ..	3·912 : 1
„ increase of the population annually	1·0167	1·00634

The population of England, according to the census of 1811, was 9,538,827, and according to that of 1821, 11,261,437, making the mean annual rate of increase of the population 1·0167. The baptisms in 1810 were 298,853, and in 1820, 343,660, making the mean annual rate of increase 1·0140.

Mr. Rickman considers the census of 1821 more accurate than

that of 1811: if therefore we suppose the ratio of the births to the population to have been constant during this short interval between these enumerations, so that the real rate of increase of the population was only 1·0140, we have 9,792,600 for the population in 1811, instead of 9,538,827, and 1,468,837 for the increase of the population between 1811 and 1821. A comparison of the registered baptisms and burials during the same time gives an apparent increase of only 1,245,000. (*See* Mr. Rickman's observations prefixed to the *Population Report*, 1821.)

Hence, if the increase was really 1,468,837, the average yearly excess of unentered baptisms over unentered burials is 22,383; and if, with Mr. Rickman, we admit the average number of unentered burials yearly to be 8,770, the average number of unentered baptisms will be 31,153. The baptisms in England, in 1820, were 328,230.

$$\frac{328{,}230+22{,}383}{11{,}261{,}437}=\frac{350{,}613}{11{,}261{,}437}=\frac{1}{32{,}044},$$

which ratio does not materially differ from that given above, in deducing which the average yearly number of unentered baptisms was supposed to be 20,696. The ratio of the population to the deaths was found by adding 8,770 to 198,634, the total of the burials in 1820; and to the marriages, by adding 191 to 91,729 (the marriages in the same year), and dividing by 11,261,437. (*See* p. 145 of the Report above alluded to.)

M. Benoiston de Châteauneuf, in the *Annales des Sciences Naturelles*, 1826, gives the following numbers as the ratio of the births to the marriages:—In Portugal, 5·14; Bohemia, 5·27; Lombardy, 5·45; Muscovy, 5·25; and in several of the southern departments of France, above 5.

In the territory of the two Sicilies, the ratio in 1828, according to the report of the Secretary of State, was 5·716 : 1.

This ratio is increased by two causes—either by the prolificness of the sex, or by the prevalence of concubinage. In the report above alluded to, the ratio of the marriages to the population is 1 : 154—in England it is 1 : 122, which difference is sufficient to account for the difference in the ratio of the births to the marriages, without supposing the former of the two causes indicated above to exist.

If the ages at which deaths take place, and the number of births, were accurately registered in a great empire, the probabilities of life would be known with the greatest accuracy, the multitude of the observations destroying any small sources of

inaccuracy; and the number of the population ($=\Sigma p_{0,n} \times b_n$) would be known far more accurately than by the laborious process of actual enumeration, for in a large district the effect of migration would be wholly insensible. It seems indeed worthy of consideration, whether it might not be possible to publish annually the bills of mortality for every parish in the empire, as is now done in London and in some great towns. If this were done, many interesting questions in science would be determined, the comparative healthiness of different districts and of different periods of the year would be ascertained, and great light might be thrown upon the efficacy of the manner in which different diseases are treated. So many questions in which property is involved are connected with the accuracy of the parish books, that it seems extraordinary that greater attention is not paid to their exactness.

No data have yet been published by which the additional premium can be determined which should be paid when the subject of the policy has any chronic disease. The only case of which I have endeavoured to determine the risk is childbirth. The deaths in childbirth during the ten years from 1818 to 1827, by the London bills, were 2,117, the number of christenings 241,352, and the number of stillborn 7,575 : which would give $\frac{2{,}117}{248{,}927}$, or $\frac{1}{117}$, for the probability that a woman does not survive giving birth to a child—making the extra premium of insurance about 17*s*. At Strasburg the deaths in childbirth are 1 in 109. At the City of London Lying-in Hospital, in 1826, the deaths were 1 in 70; in the Dublin Hospital, in 1822, there were 12 deaths among 2,675 women delivered, or 1 in 223; in the Edinburgh Hospital the mortality is 1 in 100; in the whole kingdom of Prussia, in 1817, the deaths were 1 in 112. (*See* Dr. Hawkins's *Medical Statistics.*) Most extensive returns of sickness have been furnished to the Society for the Diffusion of Useful Knowledge, by Friendly Societies, and these will no doubt furnish much valuable information upon the subject of the duration of sickness. If returns could be obtained from hospitals, of the ages at which individuals come in afflicted with different complaints, with the time they continue under treatment, and the number who die, these would also furnish the means of determining the probability of a sick person continuing sick for any given time, and the probability of an individual sick dying. From this, and the probability of an individual dying at the given age which is given by the tables, the probability of an

individual falling sick at a given age, with his *expectation* of sickness at that age, might be determined. The bills of mortality in London give the diseases by which deaths are occasioned, but unfortunately the sexes are not distinguished.

Table IX. shows the ratios of the diseases to which the deaths have been attributed at different periods in the London bills: measles seem to have increased. So little dependence, however, is to be placed on these documents, that I forbear making any further comments upon them. The column headed America is taken from the bills of mortality for Boston, New York, Philadelphia, and Baltimore; and that for Carlisle, from Mr. Milne's work on annuities.

I have also endeavoured to determine from the bills of mortality, as given in the *Annual Register* for the ten years from 1810 to 1820, the mortality and the births in London at different seasons (*see* Table XII.) The burials amounted during this period to 197,695, and the christenings to 245,287. The returns, however, are made so very irregularly, that these results, notwithstanding the very large numbers from which they are formed, are by no means accurate; for the parish clerks, as I find by examining the weekly bills, generally return the deaths and christenings of several weeks together. I have annexed observations of a similar kind given by M. Quetelet and Mr. Milne; and a table for Glasgow, which I have deduced from the bills of mortality for that city for the years 1821 to 1827. The total number of burials during that time was 31,245.

In London the mean monthly price of wheat varies very little, if at all; the same is the case with the barometer: the variation, therefore, which takes place in the number of deaths and christenings, must be principally owing to the variations in the temperature. The mean number of christenings in any month, in a given place, will also be affected by the mean time which christening is delayed after birth in that place. All the results given in Table XII. have been reduced to the radix 1200, and are corrected for the unequal lengths of the months.

I have thus endeavoured, as briefly as possible, to present the data which we now possess for determining questions connected with the duration of human life. The accordance of the results which have been deduced proves that no considerable error can obtain; for the slight difference which exists between Table VII., which I have formed from the observations at Chester, and the Table formed by Mr. Milne from those at Carlisle, is of the order

of the inevitable errors of these observations, and of the hypothesis I made with respect to the rate of increase of the population during the century previous to the observation: and in order to get rid entirely of this slight discrepancy, it would be only necessary to make the rate of increase about 1·007 instead of 1·005, as I supposed it to be. The Northampton Table, treated in the same way, would give results nearly similar.

No doubt our information on this subject will soon be much improved; for when we consider the accuracy which has been introduced into every other branch of philosophical inquiry, it appears surprising that this should have remained so far behind.

Annuities at 3 per Cent.

Age.	From Desparcieux.		From Northampton Table.	
	By approximate method.	By usual method.	By approximate method.	By usual method.
0	..	..	12·389	12·270
5	22·575	22·597	20·524	20·474
10	22·750	22·756	20·617	20·663
20	21·242	21·168	18·590	18·639
30	19·446	19·492	16·824	16·922
40	17·051	17·183	14·613	14·848
50	13·717	13·899	12·026	12·436
60	10·315	10·522	9·065	9·774
70	6·963	..	5·592	6·734

NOTE.—As the reader will no doubt infer, a considerable quantity of tabular matter accompanied this paper in the publication from which it is extracted. Had our space been less limited, we should have been glad to reprint a larger portion of it, particularly the tables illustrating the learned author's observations on the rate of increase in the population. The nature of these tables, however, will be easily gathered from the paper itself; whilst the more accurate information obtained of late years in reference to such matters deprives them, for the most part, of their otherwise intrinsic value. The registers of mortality referred to at page 282, as comprised in Table I., will be found more or less in the second volume of Mr. Francis Baily's well known work on assurances. The short table above given forms part of that referred to in the paper as Table II.—ED. *A. M.*